THREE FAMOUS PROBLEMS

1 Evaluate the probability of a "Throw Of Venus"

a) with experiments. Simulate 20 throws of four astragali by creating random integers between 1 and 10 with the calculator: .

b) by calculation.

$1 \to A$, $2 \to B$, $3-6 \to C$, $7-10 \to D$

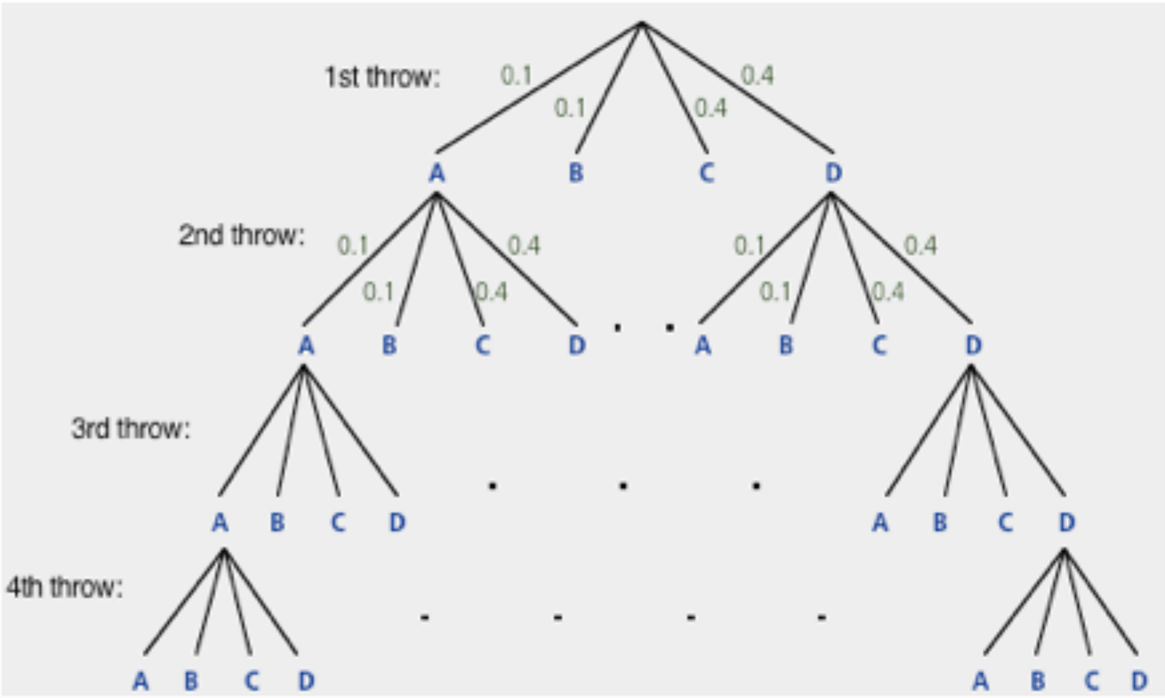
a)

throw	astragalus 1	astragalus 2	astragalus 3	astragalus 4
1	D	D	B	C
2	C	D	A	C
3	D	C	D	D
4	D	A	C	C
5	D	C	A	B
6	B	C	C	D
7	C	A	D	A
8	D	A	D	C
9	D	D	C	D
10	D	D	C	C

throw	astragalus 1	astragalus 2	astragalus 3	astragalus 4
11	D	D	B	A
12	D	A	C	D
13	C	B	C	B
14	D	C	C	D
15	D	A	A	A
16	B	C	D	D
17	C	D	D	C
18	D	D	D	D
19	D	C	C	C
20	C	C	B	D

b) by calculation.

Instead of tossing four astragali at the same time one astragalus can be tossed four times in a row. Each throw corresponds to a step in the probability tree:

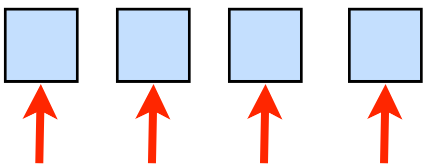


The green numbers indicate the likelihood that this branch is being taken when arrived at the branching above.
Each outcome of the experiment is represented by a path from the top to the bottom of this tree. The probability of such an outcome can be calculated by multiplying all green numbers along the path.

A "Throw of Venus" now corresponds to a path from top to bottom which contains exactly one A, one B, one C and one D, i.e. ABCD or ADBC.
So each of these paths has the same probability:

$$0.1 \cdot 0.1 \cdot 0.4 \cdot 0.4 = 0.0016$$

Further on each of these paths corresponds to a linear arrangement of the letters A, B, C and D. But how many possible linear arrangements are there?



4 3 2 1 possibilities to fill in A, B C or D.

All together there are $4 \cdot 3 \cdot 2 \cdot 1 = 24$ possibilities to arrange A, B, C and D linearly.

Finally there are 24 paths, each of them with the probability of 0.0016, so probability of a "Throw of Venus" $= 0.0016 \cdot 24 =$ **0.0384**.

sum of pips	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$\boxed{2}$ Prove theoretically Cardano's list of probabilities above.

Cardano had realised correctly that not only the number of pips are important but also on which die they appear. Let's assume that the dice are of different colour, e.g. one is red and one is blue. If the dice should be equal we colour them with a pen.
Now the sums 9 and 10 can appear in different ways:

$9 = \mathbf{3} + \mathbf{6} = \mathbf{6} + \mathbf{3} = \mathbf{4} + \mathbf{5} = \mathbf{5} + \mathbf{4}$ 4 different ways
$10 = \mathbf{4} + \mathbf{6} = \mathbf{6} + \mathbf{4} = \mathbf{5} + \mathbf{5}$ 3 different ways

So the probabilities must be:

$$p(\text{"sum of pips} = 9\text{"}) = \mathbf{\frac{4}{36}}$$

$$p(\text{"sum of pips} = 10\text{"}) = \mathbf{\frac{3}{36}}$$

3

a) Guess this number. Later on you will be able to compare your guess with the correct solution.

b) Simulate the birthday situation in a room with 30 people by randomly assigning a birthday $\in \{1, \ldots, 365\}$ to each of them.

person	birthday
1	105
2	299
3	359
4	180
5	148
6	84
7	230
8	226
9	295
10	283

person	birthday
11	338
12	228
13	122
14	166
15	294
16	317
17	94
18	262
19	295
20	171

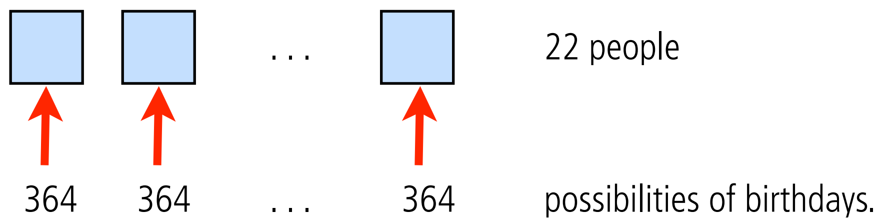
person	birthday
21	175
22	78
23	35
24	132
25	232
26	51
27	244
28	5
29	219
30	102

Person **9** and **19** share the same birthday!

$\boxed{4}$ The problem above often gets confused with the following:
How likely is it that in a class with 23 students at least one more person shares her or his birthday **with me**?

Be A = "at least one person shares the same birthday with me".
This is a very complicated event:
A = "1 person or 2 or 3 or ... share the same birthday with me".
Much simpler is the complement $\overline{A}$ = "nobody else has the same birthday as I".
What is the probability of $\overline{A}$?

There are 22 people who can have any birthday but mine:



The total of all possible outcomes: $|\Omega| = 365 \cdot 365 \cdot 365 \cdot \ldots \cdot 365 = 365^{22}$

and $\quad |\overline{A}| = 364 \cdot 364 \cdot 364 \cdot \ldots \cdot 364 = 364^{22}$

$$\Rightarrow \quad p(\overline{A}) = \frac{\text{number of outcomes in which } \overline{A} \text{ occurs}}{\text{number of possible outcomes}} = \frac{|\overline{A}|}{|\Omega|} = \frac{364^{22}}{365^{22}}$$

$$\Rightarrow \quad p(A) = 1 - \left(\frac{364}{365}\right)^{22} \approx 1 - 0.9414 \approx \mathbf{0.0586}$$

This is a very small probability and it is easy to believe. But it is exactly the probability we have in mind when we are confronted with the birthday problem. But there any other people can share a birthday and there are very many pairs, triples and so on...